Department of Mathematics

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Tutorial Sheet

Numerical Technique:

Forward, Backward and central difference operators, Interpolation, Newton's forward and backward interpolation, Lagrange's and Newton's divided difference formula for unequal intervals.

Numerical Technique:

1. Define the forward difference operator, backward difference operator, shift operator, central difference operator, and averaging operator.
2. Explain **Lagrange Interpolation** in detail and discuss its significance in numerical analysis.
3. Compare **Lagrange Interpolation** with **Newton's Interpolation**. Discuss their advantages and disadvantages. In which cases is Lagrange Interpolation preferred?
4. What is the **Newton Divided Difference** method? Explain its importance in numerical interpolation.
5. How can **higher-order interpolation** be used to improve accuracy? Explain with examples.
6. Discuss the practical applications of interpolation in engineering.
7. Prove the following relations.

   (i) $(1 + \Delta)(1 - \nabla) \equiv 1$ (ii) $\Delta\nabla \equiv \nabla\Delta \equiv \delta^2$

   (iii) $\mu^2 = 1 + \frac{\delta^2}{4}$ (iv) $E\nabla = \nabla E = \Delta$.

   (v) $(E^{\frac{1}{2}} + E^{-\frac{1}{2}})(1 + \Delta)^{\frac{1}{2}} = 2 + \Delta$

8. If u_0=1, u_1=0, u_2=5, u_3=22, u_4=57 find $u_{0.5}$
9. Given f (0) = 1, f (1) = 3, f (2) = 9, f (3) =? f (4) = 81, determine f (3).

10. Find the missing values in the following table:

x	45	50	55	60	65
y	3	-	2	-	-2.4

11. Estimate f (7.5), given f (1) = 1, f (2) = 8, f (3) = 27, f (4) = 64, f (5) = 125, f (6) = 216, f (7) = 343, f (8) = 512.

12. Evaluate y at x = 10 using Lagrange's interpolation formula, given y (5) = 12, y (6) = 13, y (9) = 14, y (11) = 16.

13. Find the third divided difference with arguments 2, 4, 9, 10 of the function f(x) = $x^3 - 2x$.

14. Use Newton's divided difference formula to calculate f(x) and determine f (3) for the data f (0) = 1, f (1) = 14, f (2) = 15, f (4) = 5, f (5) = 6, f (6) = 19.

15. Given f (0) = 1, f (1) = 2, f (2) = 1, f (3) = 10, determine the cubic polynomial and find f (4).

16. Find f(x) and Evaluate f (6) using Newton's divided difference formula given f (1) = 1, f (2) = 5, f (7) = 5, f (8) = 4.

17. Compute f (27) using Lagrange's interpolation formula given f (14) = 68.7, f (17) = 64.0, f (31) = 44.0, f (35) = 39.1, determine f (27).

18. Find the first and tenth terms of the series from the data f (3) = 4.8, f (4) = 8.4, f (5) = 14.5, f (6) = 23.6, f (7) = 36.2, f (8) = 52.8, f (9) = 73.9.

19. Evaluate f (9) using Lagrange's interpolation formula given f (5) = 150, f(7) = 392, f(11) = 1452, f(13) = 2366, f(17) = 5202, determine f(9).

20. Find the polynomial f(x) using Lagrange's interpolation formula and evaluate f (3) given f (0) = 2, f (1) = 3, f (2) = 12, f (5) = 147.

21. Find the missing term in the following table using interpolation

x	0	1	2	3	4
f(x)	1	3	9	…	81

22. Evaluate y (301) using Newton's Divided Difference Interpolation Formula for the data f (300) = 2.4771, f (304) = 2.4829, f (305) = 2.4843, f (307) = 2.4871 correct up to four decimal places.

23. Find Newton's divided difference interpolating polynomial and evaluate y (15) from the data y (0) = 30, y (6) = 48, y (20) = 88, y (45) = 238.

24. Evaluate y (35) using following data: y (0) = 7, y (10) = 18, y (20) = 32, y (30) = 48, y (40) = 85.

25. Find the cubic polynomial fitting the following data and hence evaluate f (4): f (0) = 1, f (1) = 2, f (2) = 1, f (3) = 10.

26. From the following table, estimate the number of students who obtained marks between 40 and 45:

Marks:	30—40	40—50	50—60	60—70	70—80
No. of students:	31	42	51	35	31

27. Using Lagrange Interpolation find the value of y at x = 8 given y (0) = 18, y (1) = 42, y (7) = 57 and y (9) = 90.

Ans:

8. **0.125**
9. **$Y_3 = \frac{124}{4}$ =31**
10. **2.925**
11. **421.875**
12. **$14\frac{2}{3}$ corresponding to x=10**
13. **1**
14. **10**
15. **41**
16. **6.22**
17. **49.3**
18. **Y (10) =100**
19. **810**
20. **f (3) =35**
21. **The missing term for x = 3 is y = 31.**
22. ***y* (301) = 2.4785**
23. **128.406**
24. **y (35) = 65.734375**
25. **f(x)= $2x^3-7x^2+6x+1$**
26. **The number of students getting marks between 40 and 45 = 48 – 31 = 17**
27. **68 Which is the value of y at x = 8**